SPACE AND THE LIGHT OF THE CREATION.

A NEW ESSAY IN
COSMIC THEORY

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Introduction.

MATHEMATICAL PHYSICS—and with it the entire basis of our scientific picture of the world—is undergoing an immense change. It is a well-known fact. Eminent thinkers of our time concern themselves with this question, not only from the special points of view of the mathematician or the scientist, but with regard to wider philosophic aspects. Among laymen too, in England for example during recent years, a fresh and widespread interest has been awakened. People divine that science may now be on the way to present a picture of the world, which will more truly answer to the spiritual intuitions of mankind than the scientific materialism of the 19th Century.

Yet it can not be denied, that the Physics of today is very largely groping in the dark. With symbolisms ever more abstruse as time goes on, it comes to meet the searching questions of mankind after reality. Matter as fundamental world-stuff may well have disappeared from the fields of science, but what is there to take its place? It would scarcely be too much to say that at the close of the first thirty years of our century, theoretical Physics is a science that has run to seed. Its root-ideas have grown ever farther removed from realms of experience, nay more, from anything that is accessible to human imagination. Must it continue so? Sooner or later, we may reach the insight that is necessary after all, to find the way back to the world of our experience. This is admittedly a searching question, touching the very philosophy that underlies our Science. In fact, it has to do with the entire contrast of the orthodox Newtonian School, which in a wider sense is dominant in Physics to this day, and of that other tendency in modern spiritual history which arose for a short time among the poets and natural philosophers of the early 19th Century, and came to expression above all in the life and work of GOETHE. To speak in the Goethean spirit, the purpose of scientific thinking is not to spin out complicated theoretic systems farther and farther removed from experienced reality, but to fertilise within the human soul—out of those spiritual sources which are accessible to thought—what the soul has experienced by contact with the world of Nature. The human soul evolves in this process—grows to a higher level—and as it does so the circle of direct experience increases. The sphere of knowledge grows inasmuch as the shackles of imperfection and the veils of blindness fall from the soul. The soul experiences increasingly and yet more tenderly and richly by contact with the world of Nature—Nature including, we need scarcely add, that which we ourselves bring forth in her by experiment and ordered observation.

Such was the method to which GOETHE remained true in his prolonged and many-sided scientific researches. In our own times RUDOLF STEINER shewed by his whole life's work that the same method can also be pursued in harmony with subsequent developments of Science. Nay more, he shewed how as we do so Science will grow into a conscious experience of the spiritual source and origin of things,—no shadowy or abstract 'spiritual source' remotely apprehended by philosophic speculation, but alive and present, and filled with abundant content.*

The following is an attempt to shew, how by this Goethean and at the same time Anthroposophical method we may also come upon a true foundation for Mathematical Physics, one that appeals to our sense of the realities of life now that the old materialist foundation has disappeared. As we set out upon this path we shall unite at the same time with a most essential stream in the spiritual history of recent times, namely the modern Synthetic Geometry and the higher Algebra, which is so intimately connected with it. Here too, as in other spheres, it will be proved that Anthroposophy is no back-water in the stream of modern life, but on the contrary is able to unite within it the most fertile seeds of science and research. It is as though the real spirit of our time, which is at work in manifold directions, had come to conscious life in Anthroposophy.

It scarcely need be said that we are well aware of the imperfections of this attempt. Added to that, the treatment of the subject in the present essay is necessarily brief and sometimes aphoristic. A fuller presentation will be given in a book which is now in preparation.

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*Compare Goethe's essay, so often quoted by Rudolf Steiner, on the experiment as mediator between object and subject (*Anthroposophy* Quarterly, Easter 1932).

CHAPTER I.

Radiation of Space.

AMONG the scientists and thinkers of the 19th Century, those who created the new method in Geometry, such men as PONCELET and CHASLES in France, the Swiss geometrician JACOB STEINER, CAYLEY and SYLVESTER in England, VON STAUDT, FELIX KLEIN and others, pursued a rather lonely path, though not unrecognised by their contemporaries. The physicists of the prevailing school combined the mathematical instrument of thought in a rather impure way with imaginations borrowed from the world of the senses; imaginations as of atoms, ether-waves, etc. These men, meanwhile, were bringing forth in the pure realm of thought quite new discoveries, wherein the spirit of the new age was working. Thus RUDOLF STEINER in the early eighties, at the very time when he related his new light of knowledge to the scientific work of GOETHE, perceived the high significance of the new Geometry for a more spiritual idea of Space, and for a world conception free from the bonds of naïve materialistic fancy. Here is indeed an inner connection very significant of the true spirit of the time. We can perceive a deep relationship between the thought-forms of the new Geometry and the 'Theory of Metamorphosis' which GOETHE discovered in the world of plants. The ancient, classical Geometry—in the form in which it entered, through DESCARTES above all, into our time—is bound to rigid and once-given forms. Moreover, in its method it is atomistic, additive. It regards the *point*, above all, as the given thing; straight *line* and *plane* are treated more or less as derived, composite entities, as aggregates of points. Not so the newer Geometry; it does not take its start in this one-sided manner from the point. Point and plane, for this Geometry, are equally valid as the fundamental entities of Space; the one no less original and basic, no less single than the other. In the true spirit of modern Geometry, the plane has an equal right with the point to be treated as an individual and primary thing. Yet the two kinds of entity, point and plane, are so related to each other in the community of Space that each can be thought of as composed of a multitude of the other. The points of a plane are indeed 'parts' of the plane, if we treat the latter as an aggregate of points. We are free to do so, but if we do, the spirit of Geometry obliges us on the other hand to recognise the point itself as composite, and the doubly infinite number of *planes* which contain it as *parts of the point.*

Between the two kinds of entities—plane and point—there is a third, the straight line, which in a manner of speaking mediates

between them. It is the entity which rays through space, weaving the myriad forms with which space is pregnant. The line* itself can be regarded in three ways: (1) as a single, individual being; (2) as an aggregate of all its points; or (3) as an aggregate of all its planes. If we describe the points of the line as its several parts, here once again, pure geometrical thinking will oblige us also to recognise the *planes* of the line—that is, the planes which have in it, their common axis—as *parts of the line* (Figure 1).

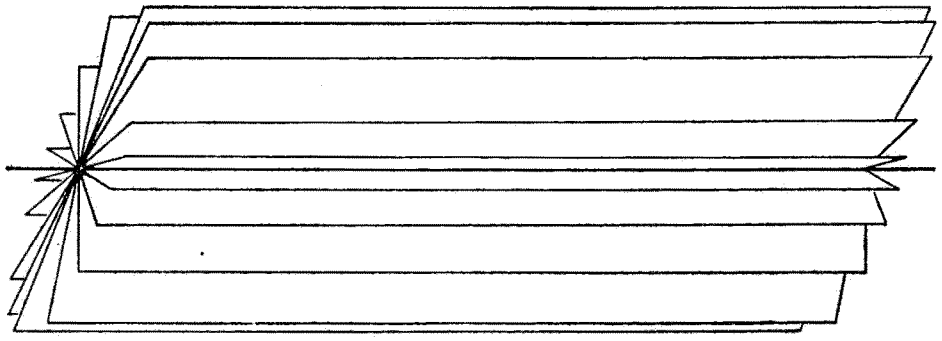

Figure 1. The line in its twofold aspect (physical—ethereal) : as a manifold of points and as a manifold of planes.

Modern Geometry here leads us to a kind of 'archetypal concept' for the world of Space. This concept plays a fundamental part in the Anthroposophical understanding of Nature. We refer to the polarity of the 'physical' and the 'etheric' or 'ethereal.' The 'self-evident' axiom of Euclid which we learned at school, whereby the whole is always greater than the part, is the true aspect of the physical—or, we might say, of the *real existence* in the midst of which we live on earth. The paradox, latent if not expressed in the axioms of modern Geometry, whereby the whole (the point for example, considered as an aggregate of planes) can be smaller in extension than any one of its parts, is the true aspect of the ethereal—or, we might say, of that which works creatively out of the cosmic periphery—of that which is *becoming* and not yet *become* in earthly Nature. Rudolf Steiner told how for clairvoyant vision an etheric organ—for example, the etheric heart or liver—does indeed appear smaller than any of its parts, smaller, that is to say, than the cosmic irradiations which compose it in its ethereal totality. Here, of a truth, the very opposite is 'obvious' than in the physical aspect.

*Here and throughout the following pages, the single word 'line' is invariably used to mean 'straight line.' It should be noted that when we speak of a straight line in modern Geometry, we always think of the line in its ***entirety***—extending to the infinite in both directions. Likewise a plane is always thought of as extending to the infinite in all directions. Only in this way can the line or the plane appear to us as an organic whole.

Along with this well-known 'Principle of Duality'* as between point and plane, we have the fundamental operation of the new Geometry, commonly known as the method of 'projection and section.' We should prefer to call it by a more living name. It represents, as we shall see, in the realm of geometrical imagination, a kind of germinating and re-creative process. Modern Geometry originated, in fact, out of the newly awakened activity of the human eye in the 'age of the Spiritual Soul.' It arose from the freshly awakened experience of spatial perspective among the artists, scientists and explorers at the beginning of modern time. The practical art of perspective grew by degrees into a pure science which describes the radiant, perspective, light-filled nature of Space itself. Thus there arose what is now known as 'Projective Geometry.' We refer for example to DESARGUES and PASCAL in the 17th Century. Side by side with this there were the constant researches into a knotty problem of the classical Euclidean Geometry, to wit, the axiom of parallels (SAVILE, WALLIS, SACCHERI, LAMBERT and others.) With these researches, as will appear in the sequel, modern humanity, at the very time when the old traditional knowledge of the universal Ether grew chaotic and confused, was preparing to approach once more, though in another way, the secret of the cosmic sphere, the cosmic periphery with its etheric potencies.

The individual Ego of man upon his earthly stance freely looks out into the wide surrounding spaces. Such was the inner experience of the new time; it found its reflection in the art of the Renaissance. Projective Geometry, or the Geometry of radiant metamorphosis, which grew to maturity—during the lifetime of GOETHE—in the works of PONCELET and JACOB STEINER among others, was only the other aspect of this new stream in spiritual life. The clear and single forces of the eye are active in the thought-forms of the new Geometry. The same activity, wherewith the eye beholds in Sun-filled Space the pictures of the world, becomes creative in pure thought. A given spatial picture is received by straight lines or rays into a point, as into an eye

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*The 'Principle of Duality,' in its most general form, as here intended, is contained in the simple statement that the original relations of the three kinds of entity—*point*, *line* and *plane*—are entirely symmetrical with respect to 'point' and 'plane.' Therefore, in every theorem of Projective Geometry, dealing with points, lines and planes, and with the forms that arise from these, the mere interchanging of the concepts *point* and *plane* will result in a theorem no less significant than the first, and, moreover, such that the truth of each theorem is implied in the truth of the other. Each of the two theorems is then said to be the 'dual' of the other—the word 'dual,' in this connection, signifying a polarity or polar antithesis.

As an example we may state the axioms, self-evident to our imagination:—

Any two *points* of Space have a single *line* in common, namely, the straight line which 'joins' them.

Any *three points* of Space, not in a line with one another, have a single *plane* in common.

Any two *planes* of Space have a single *line* in common. (If the planes are parallel, this line is 'at infinity.')

Any *three planes* of Space, not in a line with one another, have a single *point* in common.

The form-of-rays which thus arises in the point is called a 'cone,' in the wider, mathematical sense of the term. To every point of the spatial picture there corresponds a line or ray of the cone; to every line, a plane of the cone. The cone, consisting as it does of so many lines and planes in a single point, is no longer outwardly visible to the imagination; at most it is so in the simplest instances, and even here we easily delude ourselves. It is not really like an outward form or picture that confronts us; but it contains, as in a seed or germ, the entire form from which it was derived. If we set over against it any other plane of space, the form is reproduced or 'projected' in this plane, and there arises a fresh picture—a metamorphosis of the picture originally given. Moreover this is possible in every variety of metamorphosis, according to the relative positions of the picture-plane and of the 'germinating point,' if we may call it so. By repetition of this basic operation, Projective Geometry studies the radiant metamorphoses—'linear transformations,' as they are generally called —of geometric forms, which it thus recognises in their deeper essence, in their essential type or archetype.

Entering actively and by constant practice into these forms of thought, we realise increasingly, not as a mere fortuitous analogy, but as penetrating into the very essence of the matter: *This metamorphosis by means of 'germinating point' or 'eye,' and 'picture plane,' which we here follow in pure thought, contains the same creative principle of Space which Goethe recognised as a fundamental law of Nature in the life of plants.* GOETHE described it as a rhythmic interplay in the polarity of expansion and contraction. In the living plant we have the seed, the germinating point or 'eye,' over against leaf. In the pure thought of our geometry we have simply 'point' and 'plane.' Just as the plant contracts its archetypal form into the tiny seed or eye where it is not apparent to external sight, only to bring it forth again in a new time or environment, in corresponding metamorphosis, in the outward and visible form of the leaf; so in projective geometry an archetypal form, once given, is ever and again received as a germ into a fresh eye-point and reproduced in a fresh plane. And in this process there is engendered every conceivable metamorphosis of the original form, which as a faithful type or archetype passes unchanged yet changing through the many forms of its appearance. Just as the mathematical eye-point centres within itself, as in a cone, the form of lines and planes raying into it from the periphery, so does the plant-seed, concentrated as it is towards a single point, fulfil its function in external Nature. It is no physically self-contained or finished 'thing;' it is an eye-point, fertile to receive the archetypal form of its species, raying in upon the waves of the cosmic Ether from the multiform periphery of the great Universe.

The study of these radiant or linear transformations of Space during the 19th Century gave rise for the first time to a true and evident systematics of the many single forms of Geometry. Thus did Projective Geometry—precisely the Geometry of these trans-

formations—become the very basis of all Geometry. In the classic phrase of ARTHUR CAYLEY: Projective Geometry is *all* Geometry. Not only did the metamorphoses of once-given forms become intelligible in this way. By a progressive enhancement of the radiant process, the very same principle of action led from the archetypal entities of Space—point, line and plane, which in themselves are well-nigh formless, void of all special form—to the creation of ever higher types, as it were out of the nothingness of empty Space. Hereof the simplest example: the purely projective construction of cones and conics after JACOB STEINER. Thus did Projective Geometry become 'Synthetic Geometry' in the spirit of modern Time. (The classical, Euclidean Geometry was itself 'synthetic'; but the spirit of the Time which inspired it no longer works creatively for the modern mind.)

In the light of RUDOLF STEINER'S Anthroposophical researches we recognise the inner reason for this deep harmony between the newer Geometry and the Goethean science of the Metamorphosis of Plants. The plant, in fact, as RUDOLF STEINER shewed, has quite another relation to the creative forces of the world than the mineral kingdom. The plant, with its rhythmic life in time, has a more intimate relation to the creative forces of the Sun. In RUDOLF STEINER'S terminology of Spiritual Science, the plant has an 'etheric body' of its own. This signifies, in effect, that the plant partakes in the Space-creative, Space-radiating forces of the spiritual cosmos. A mineral is a mere 'thing among things' within the world of Space. Not so the plant, in so far as it is alive; it grows in Time, in the cosmic solar rhythm of day and night, summer and winter, planets and constellations in their courses. *Living in Time, it creates its own Space from within.* In the unfolding of the plant we behold with eyes of sense a process which in the Universe is supersensible, an actual *creation of Space out of pure realms of Time.** The Sun-ether of the Universe is there at work, individualised as it were within each single plant. Hence we can never succeed in explaining the essential form of a plant out of external, spatial causes, after the pattern of an inorganic science. External causes may no doubt explain its various modifications and adaptations; what is *essential* in the form of the plant springs from a supersensible entity, from a Space-creative archetype which hiddenly indwells it.

Qualitatively, we find again in the plant the same essential trinity which we behold in the mind's eye as the fundamental trinity of space—point, plane and line. The seed or germ, the 'eye' as it is called in the old Botany, partakes of the nature of the point; the leaf, of the nature of the plane. The life and growth of the plant takes place, as we have seen, in the rhythm

*A hitherto unpublished lecture by Rudolf Steiner (Whitsuntide lecture at Dornach, June 4, 1924) contains what is essential to an understanding of these matters. See also his early work, ***Theory of Knowledge for Goethe's Conception of the World***, on the comparison of 'organic' and 'inorganic' science, to wit: Biology and Physics, and the organic, that is to say, the cosmic, aspect of the latter.

of expansion and contraction: expansion into the plane of the leaf, contraction into the germinating point. All this takes place by way of radiating line-forms, as revealed in stalk and stem and in the inner network of the leaves. Here we discover, among the very fundamentals of plantgrowth, the phenomenon known as '*anastomosis.*'* This also has its counterpart in the basic laws of pure Geometry. The plane itself, for example—seen from a certain aspect—owes its existence to an 'anastomosis,' as it were, of the lines in Space. Some modern authors, who do not count the plane, as we have done from the very outset, among the elemental entities of Space, include the following or an equivalent statement among the axioms of Geometry.

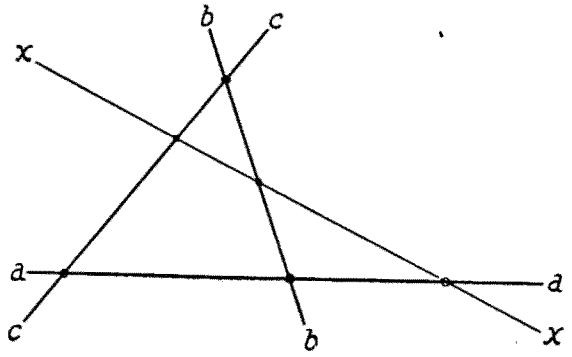

Figure 2.

'If three lines of space (a, b and c, Figure 2) meet one another in pairs in three distinct points (bc, ca and ab), then every other line of space (x) which has distinct points (bx and cx) in common with two of the lines (b and c) will also have a point (ax) in common with the third.'

The lines (x) of space, which satisfy these premisses, actually weave the plane which is uniquely given by the three points (bc, ca and ab). It is a kind of anastomosis in the realm of pure Geometry. And the same principle reveals itself in a yet higher form.

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*Anastomosis is the merging into one another of the 'veins' of a leaf, so that they form a continuous and closely woven surface-network: the leafy 'skeleton' which we so often see—dead relic of a living process—in the winter time.

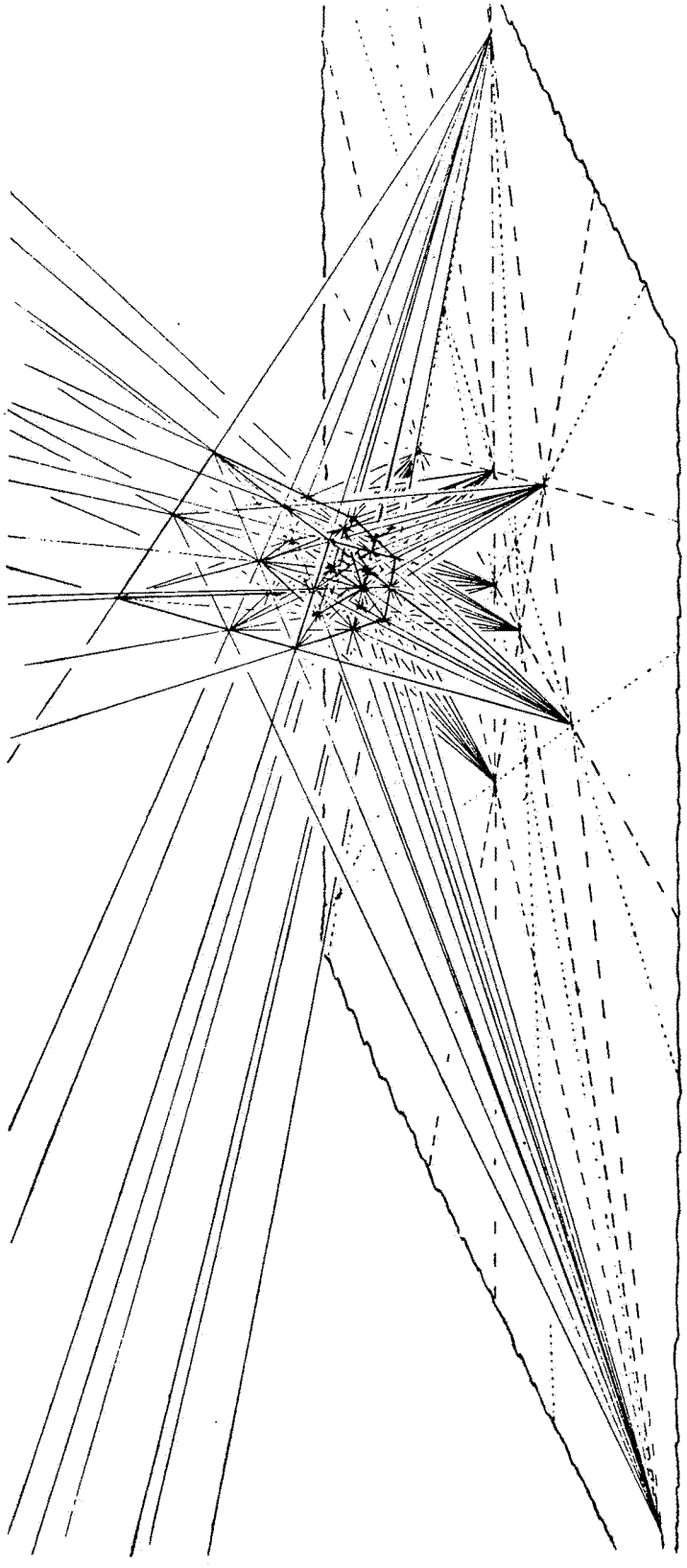

Figure 10.

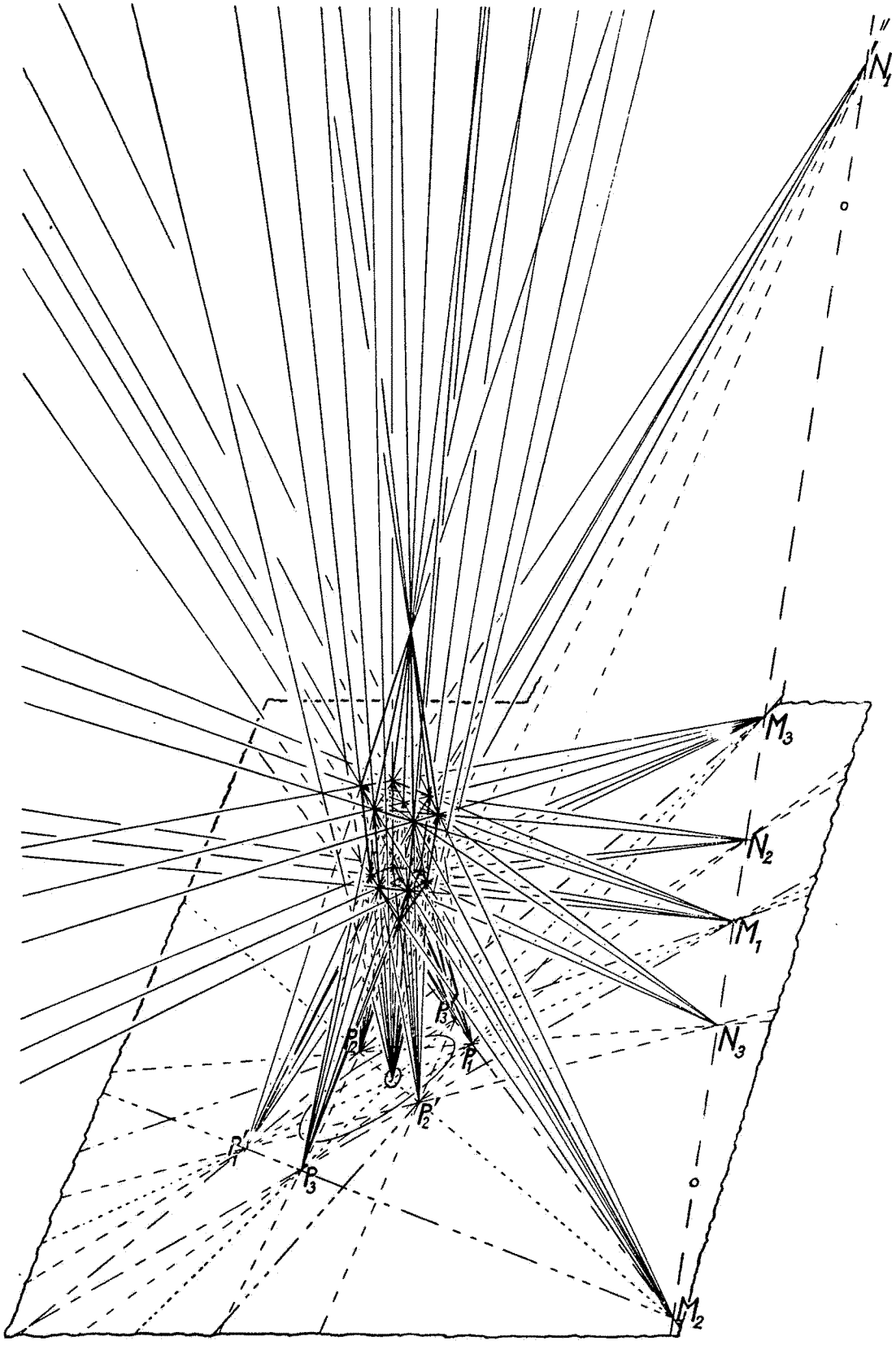

Figure 19.